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TECHNICAL NOTE

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EFFECTS OF SOME TYPICAL GEOMETRICAL CONSTRAINTS ON LUNAR TRAJECTORIES

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SUMMARY

A study has been made to determine the effects on lunar trajectories of some typical geometrical constraints. The constraints considered in this study are of two types, those resulting from specification of the trajectory characteristics near the earth and those associated with the specification of the approach conditions at the moon. The effects of the constraints are discussed from the standpoint of the limitations imposed by the constraints on the possible launch days during the month and also on the possible launch times during the day for three types of launch trajectories: direct-ascent, coasting-orbit, and parking-orbit launches. Application of the various constraints individually or in combination seriously restricts the allowable launch times during the month and day for the direct-ascent launch; whereas, less serious restrictions result for the coasting- and parking-orbit launches.

INTRODUCTION

If there are no restrictions on the launch conditions of a vehicle or on the conditions of approach to the moon, then there are no limitations on the time of the lunar month at which the vehicle can approach the moon. Practical considerations, such as launch-point location, missile-range safety, accuracy tolerances, guidance and navigation, and the limited allowable variations in the injection conditions, impose restrictions on the lunar declinations which can be accommodated. It is of interest to examine the limitations on the possible launch times of lunar missions that are imposed by some of these restrictions.

A typical design parameter for lunar missions is the specification of the lighting conditions on the surface of the moon, that is, the phase of the moon. For example, oblique lighting is desirable for photographic determination of the surface features of possible landing sites, instrumented soft landings would be designed to utilize solar power during a full lunar day, and landing sites near the terminator might be desirable for short-period manned missions so that some control can be obtained over environmental conditions. For a particular year, the declination of the moon at a given phase of the moon varies between maximum and minimum values which correspond approximately to the mean inclination of the moon's orbit to the equator for that year. This variation of declination at a given phase is primarily due to the difference between the synodic and sidereal periods of the moon. Because of this variation, it is of interest to analyze the capability for placing vehicles in the vicinity of the moon at all possible lunar declinations.

SYMBOLS

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Refer to figure 1 for an illustration of some of the angular parameters defined.

e — eccentricity

i — inclination of moon's orbit to earth's equator

r — geocentric radius

v — true anomaly

$\frac{V}{V_p}$ — ratio of injection velocity to local parabolic velocity, $V_p = 35{,}384.5$ fps at altitude of 300 statute miles

α — initial angle between the earth-moon and earth-vehicle planes, positive when vehicle approaches moon from north

β — launch azimuth angle, positive east of north

γ — injection angle, angle between injection velocity vector and local horizontal

δ — declination of moon at closest approach of vehicle

λ — latitude

η — heading angle of moon at closest approach of vehicle to moon

θ — difference in angular positions of moon and launch point, $\theta = \phi_n - \phi_L$

ϕ — angular position measured in earth equatorial plane eastward from moon ascending node

ρ heading angle of vehicle at closest approach of vehicle to moon

ψ geocentric angular travel

ψ_b geocentric angular travel between injection and closest approach to moon

ψ_r reduced geocentric angular travel, $\psi_s - 2\gamma$

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ψ_s geocentric angular travel from launch point to injection point

ψ_t total geocentric angular travel between launch point and moon, $\psi_b + \psi_s$

$$\Delta\psi = \psi\left(\frac{V}{V_p}\right) - \psi(0.995)$$

Subscripts:

i injection point

L launch point

m moon

ANALYSIS AND DISCUSSION

General Considerations

For the complete determination of lunar trajectories it is necessary to specify a number of conditions which can be conveniently separated into dynamical conditions in the plane of the trajectory and geometrical conditions which define the orientation of this plane with respect to the earth-moon plane. The parameters which specify these conditions are not all independent and if restrictions are imposed on several of these parameters there may be resulting restrictions on the remaining ones. This study was initiated to determine the limitations on the allowable declinations of the moon resulting from restriction on some of the geometrical parameters.

The orientation of the trajectory plane is determined by the launch azimuth, the latitude of the launch point, and the declination of the moon at the closest approach of the vehicle. The dynamical parameters

are the earth-moon distance and the injection conditions: time, velocity, flight-path angle, and geocentric radius. Of these five parameters only the time depends directly on the geometry; that is, the launch time must be chosen so that the geometrical constraints are satisfied. As for the other dynamical parameters, the earth-moon distance varies only 5 percent from its mean value and the injection velocity, angle, and geocentric radius are usually determined by payload, guidance requirements, or other conditions not directly related to the geometrical parameters. In the analysis the launch point is considered to be at a latitude of 28.5° north. The launch azimuth angle is to be between 40° and 115°, as specified by Atlantic Missile Range safety requirements. L
Injection into the ballistic trajectory is assumed to take place at an 1
altitude of 300 statute miles (geocentric radius of 4,260 statute miles), 5
with injection angles between 0° and 30° and injection velocity ratios 1
between 0.992 and 1.02; these values correspond to earth-moon flight 7
times of about 73 and 33 hours, respectively. The earth-moon distance is taken to be the mean value of 238,857 miles and the inclination of the earth-moon plane to the equatorial plane is taken as 23°, a value which corresponds approximately to the actual inclination during the year 1963.

In order to develop analytical relations between the various parameters which must be specified for a lunar trajectory, in the present analysis use is made of some simplifying approximations. In order to determine values of the dynamical parameters, it is assumed that the injection conditions for the ballistic trajectory from the earth to the moon can be calculated with sufficient accuracy from the two-body equations which neglect the gravitational attraction of the moon. Reference 1 indicates that this assumption is valid for obtaining good first-order estimates of injection conditions for three-dimensional lunar impact trajectories. Other types of lunar trajectories can be generated from these impact trajectories by making small changes in the initial conditions. By making use of this approximation, the geocentric angular travel of the vehicle from the injection point to the moon can be calculated from equations (A1) to (A4) in the appendix. Typical values of the angular travel are given in the following table for three injection angles and an injection velocity ratio of 0.995:

γ, deg	ψ_b, deg	$\psi_b + 2\gamma$, deg
0	169.8	169.8
15	139.9	169.9
30	110.7	170.7

In this table it is seen that the sum $\psi_b + 2\gamma$ is approximately independent of the value of the injection angle. It is a characteristic of nearly parabolic orbits that this sum depend only on the injection velocity ratio. This property of the motion will be utilized in a later section to introduce an angular parameter which will make the results independent of the injection angle.

Ascent Trajectories

Three types of launch techniques are considered in the study: a direct-ascent injection into the ballistic trajectory, injection following a coasting period before the final burning phase, and injection following several revolutions in a parking orbit before the final burning phase.

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A direct-ascent launch is defined here as one for which the lengths of the coasting periods between successive stages are determined by aerodynamical, structural, and/or dynamical considerations and are not determined directly by considering the relative positions of the launch point and the moon. For this case the total geocentric angular travel from the launch point to the injection point is essentially the burning arc of the booster.

After the launch point is specified, the declination of the vehicle after it has traversed an arc ψ_t is determined by the heading angle at launch. Conversely, the heading angle required to approach the moon at a given declination is a function only of that declination and total angular travel from the launch point to the moon. For a velocity ratio of 0.995 and the injection angles presented in the preceding table, the allowable declinations of the moon can be calculated as a function of the heading angle β and the angular travel from the launch point to the injection point ψ_s. The analytical relations between these parameters are given by equations (A5) to (A10) in the appendix. The results of these calculations are shown in figure 2, where the location of the moon is specified by the angle ϕ_m measured eastward from the moon's equatorial ascending node as shown in figure 1. Figure 2(d) gives the moon's declination as a function of its angular position. For direct-ascent launches the burning arc, that is, the arc from launch to injection, is not expected to exceed 60° or about 4,000 miles over the earth's surface. Figure 2 can be used to obtain approximate initial conditions for lunar missions. For example, consider a launch system with a burning arc of 20° and a resulting injection angle of 30°. If the mission requires that the vehicle is to approach the moon at its decending node $\left(\phi_m = 180^\circ\right)$, figure 2(a) indicates that a launch azimuth of 65° will be required.

A coasting- or parking-orbit launch is defined as one for which the vehicle is first placed in a nearly circular orbit about the earth, then at a predetermined angular travel from the launch point a velocity increment is added to satisfy the injection conditions. For this type of launch, the geocentric angular travel in the coasting orbit is determined directly by considering the relative positions of the launch point and the moon. For the purpose of identifying the length of the arc, a coasting orbit is one for which the total angular travel before injection is less than or approximately equal to 2π radians, while a parking orbit is one for which the angular travel is greater than 2π radians.

As shown in the preceding table, the sum $\psi_b + 2\gamma$ can be considered independent of the value of the injection angle. The total geocentric angular travel $\psi_b + \psi_s$ from the launch point to the vicinity of the moon depends only on the geometrical parameters; therefore, this sum is also independent of the injection angle. Consider the identity

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$$\psi_s - 2\gamma \equiv \left(\psi_b + \psi_s\right) - \left(\psi_b + 2\gamma\right)$$

Both terms in parentheses have been shown to be independent of the injection angle; therefore, the difference on the left must be independent of the injection angle. If the results are expressed in terms of a new function, the reduced angular travel $\psi_r = \psi_s - 2\gamma$, the curves of figures 2(a), (b), and (c) reduce to a single plot.

Figure 3 shows the variation of reduced angular travel over one full cycle and, like figure 2, can be used to obtain approximate initial conditions for lunar trajectories. As indicated, figures 2 and 3 are for a velocity ratio of 0.995. The figures can be used at other velocity ratios by considering the difference $\Delta\psi$ between ψ_r or ψ_s at the desired velocity ratio and at a ratio of 0.995. The difference can be calculated from the two-body equations given in the appendix. The results of these calculations are presented in figure 4. Corrections from figure 4 shift the curves of figures 2 and 3 vertically along the ψ-scale. For the velocity range considered, the maximum change in ψ is about 20°.

Constraints and Their Effects on Lunar Trajectories

The constraints considered in this study are of two types, those resulting from specification of the trajectory characteristics near the earth and those associated with the specification of the approach conditions at the moon. The effects of the various constraints are discussed from the standpoint of the limitations imposed by the constraints on the possible launch days during the month and also on the possible launch times during the day for the three types of ascent trajectories discussed in the previous section.

Constraints associated with the near-earth trajectory characteristics.- Choosing a launch site imposes certain restrictions on the trajectory; for example, it defines particular values of the latitude and longitude of the launch point. If a number of facilities were available for the launching of lunar vehicles, it would be of interest to determine the effects of launch-point location on possible lunar trajectories; however, lunar missions of interest here are expected to be launched from Cape Canaveral, Florida. Therefore, in this study the latitude and longitude of the launch point will be considered to be the constant values for Cape Canaveral.

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There is an additional restriction associated with most launch sites resulting from the requirement that the vehicle not pass over populated areas during the initial boost phase. The Atlantic Missile Range safety requirements resulting from these considerations are that the launch azimuth be between the approximate limits of 40° and 115° east of north. This restriction is illustrated in the figures by the hatched regions. Figures 2(a), (b), and (c) illustrate the effects of these requirements on direct-ascent trajectories. To demonstrate the use of the figures, consider a lunar vehicle with a booster burning arc of about 20°. The plot shows that, for $\psi_s = 20^{\circ}$ and $\gamma = 30^{\circ}$, the allowable range in lunar position excludes angles from $\phi_m = 32^{\circ}$ to $\phi_m = 148^{\circ}$ or declinations greater than 12°, and thus restricts lunar missions to about two-thirds of the month. As the injection angle is decreased the restricted period is seen to increase to the extreme case of no possible missions for a zero injection angle. Thus for direct-ascent launches the range azimuth limits can greatly restrict the number of possible launch days during the month.

Since a correction from figure 4 for a change in injection velocity shifts the curves in figure 2 vertically along the ψ-scale, an appropriate change in the velocity makes it possible to extend the range of lunar declinations which can be accommodated for a direct-ascent launch. However, the injection conditions are approximately determined by payload, guidance, heating, and/or other considerations and any appreciable change will result in some penalties. Therefore, changing the injection conditions will not be considered as a practical means of appreciably increasing the launch time capability.

Figure 3 can be utilized to demonstrate the effect of the range azimuth limits on coasting-orbit launches. The same results can be made to apply to the parking-orbit launch by increasing the reduced angular travel by 360° times the number of complete revolutions before injection. Within the azimuth range permitted, there are two bands of coasting arcs for which there are no restrictions on the time of the month at which a lunar vehicle can be launched. These bands occur at reduced angular travels of about 120° and 300°. Again, consider as an example a vehicle with a total burning arc of 20°; the coasting arc in circular orbit can be calculated and the results give a short and a long coast for each injection angle as shown in the following table:

γ, deg	Coasting arc, deg	
	Short coast	Long coast
0	100	280
15	130	310
30	160	340

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By designing the launch trajectory with a fixed coasting period and by changing the launch heading angle throughout the month, it is possible to initiate lunar missions on any day of the month. It is expected that utilizing such a procedure will result in, at most, a small payload penalty, since the payload depends indirectly on the launch heading angle through the earth's rotational velocity, which is only a small part of the required injection velocity for lunar missions. In summary, it appears that the range azimuth safety requirement may seriously restrict the time of the month during which direct-ascent lunar missions can be initiated. However, by designing the launch trajectory with a constant coasting arc it is possible to eliminate this restriction.

Constraints associated with the near-earth trajectory characteristics often result from the utilization of a particular type of launch trajectory. For example, parking orbits are of interest because of some relative advantages of these orbits as compared with the direct-ascent or coasting orbits. For manned missions, some advantages appear from rendezvous, guidance, navigation, system check-out, and mission abort considerations. Some of these relative advantages present limitations on the launch conditions. For example, reference 2 indicates that, for efficient rendezvous, the inclination of the parking orbit to the equator should be approximately equal to the latitude of the launch point. If both the ferry vehicle and orbiter are launched from the same site, then for efficient rendezvous the launch azimuth of both vehicles must be nearly due east. Figure 3 indicates that if the ability to approach the moon on any day of the month is to be maintained while launching nearly due east, the coasting arc in orbit must be varied throughout the month. Thus when the heading angle is specified the constant-coasting-arc trajectories discussed previously are not possible. It should be noted that in figure 3 the precession of the parking orbit due to the earth's oblateness is not considered. If the time in orbit does not exceed a few orbital periods this precession will not produce any large differences in the launch azimuth and reduced angular travel as compared with those given in figure 3.

<u>Constraints associated with approach to the moon</u>.- The second class of constraints to be discussed are those associated with the specification of the approach conditions at the moon. The nature of the constraints will depend on the particular mission to be accomplished. For example, a requirement for direct-descent trajectories to the lunar surface may be that the vehicle land on the visible side of the moon. For a circumlunar mission, the minimum distance from the moon and the orientation of the selenocentric hyperbola must be specified if the vehicle is to reenter at the correct point on the earth. Similarly, for most efficient establishment of lunar satellites, the elements of the selenocentric approach hyperbola must be chosen so that the periselenian will be at the desired orbital altitude and the orbit will have the desired inclination.

For highly inclined selenocentric orbits the previous conditions can be satisfied by making small adjustments in the launch azimuth and launch time from the nominal values given by the two-body analysis. To establish low-inclination selenocentric orbits, without an excessive expenditure of rocket fuel, requires the specification of an additional important parameter - the initial angle between the vehicle trajectory plane and the earth-moon plane. Minimizing the initial angle between the trajectory plane and the earth-moon plane aids in establishing a low-inclination lunar orbit while it does not seriously restrict the establishment of high-inclination lunar orbits. This angle is also important from guidance considerations. As indicated in reference 1, guidance requirements for lunar impact or for establishing a lunar satellite become more stringent as the angle between the planes increases. Similar results are expected for circumlunar missions.

The initial angle α between the vehicle trajectory plane and the earth-moon plane is a function of the angular position of the moon ϕ_m and the launch azimuth β and can be calculated from equations (A8) to (A12) in the appendix. The results of these calculations are presented in figure 5 for the assumed earth-moon geometry and for launch azimuths within Atlantic Missile Range safety limits. It can be seen by comparing figures 3 and 5 that the minimum angle between the planes for any position of the moon occurs for a due east launch, a result derived in reference 3. Thus, the condition for minimizing the inclination of the trajectory plane to the earth-moon plane is the same as the condition for assuring efficient earth-orbit rendezvous as discussed in the previous section. This is an especially advantageous situation if the purpose of the mission is to establish a lunar equatorial orbit, for this condition nearly minimizes the impulse required at the moon to establish the orbit. The absolute minimum angle between the planes is 5.5° and occurs when the moon is at an equatorial node. Within range azimuth safety limits the maximum angle between the planes of 78.5° occurs at the same lunar position and a launch azimuth of 40°.

Comparing figures 3 and 5 shows that some of the fixed short- and long-coast launches discussed previously are unsuitable from the standpoint of obtaining a small angle between the planes; for if the moon is near the ascending node $\left(\phi_m = 0\right)$, the long-coast launch produces angles between the planes of about 50°. Similar values are obtained for the short coast if the moon is decending $\left(\phi_m = 180^{\circ}\right)$. However, the ability to launch on any day of the month can be maintained, while keeping the angle between the planes at a minimum, by utilizing a variable coasting arc in conjunction with a nearly due east launch azimuth. The coasting arc is taken to be a long coast when the moon is descending and a short coast when the moon is ascending. By utilizing this procedure, the angle between the plane can be reduced to less than 20° throughout the entire month. L1517

Restrictions on Launch Time

The required angular position of the launch point can be calculated from the geometrical relationships in the appendix. Figure 6 illustrates the variation of ϕ_L over a period of 1 lunar month for values of reduced angular travel which give launch headings within the range of azimuth safety limits as indicated by the hatched regions. The angular position of the launch point defines the daily launch time, for in 1 day the launch point rotates through 360°. As an example of the use of figure 6 to obtain approximate initial conditions, consider a launch trajectory with a burning arc of 20°, a coasting arc of 50°, and an injection angle of 15°; for this system $\psi_r = 40^{\circ}$. If the vehicle is to approach the moon at maximum negative declination $\left(\phi_m = 270^{\circ}\right)$, the required launch-point location is about $\phi_L = 60^{\circ}$. Figure 5 shows that the angle between the planes would be about -18° and figure 3 gives a launch azimuth of 97°.

The preceding sections have presented the restrictions imposed on the possible launch days of the month by constraining or specifying some geometrical parameters. Throughout the analysis it is implicitly assumed that the position of the launch point will satisfy the equations given in the appendix for any position of the moon. However, the launch point is fixed on the rotating earth with the moon revolving about the earth and this relative rotation imposes an additional restriction on the launch time. This constraint can be expressed by

$$\phi_L = 27.3\phi_m + \text{Phase angle} \tag{1}$$

where ϕ_L is the angular position of the launch point at launch as measured in figure 1. The coefficient of the first term on the right is the ratio of the moon's orbital period to the earth's rotational period. The phase angle is a function of the time from launch, the launch date, and the total flight time. The explicit dependence on the time from launch produces a small periodic variation due to the eccentricity and obliquity of the moon's orbit. If this variation is neglected, the phase angle can be considered to be a constant. It is of interest to examine the restrictions imposed on lunar missions by this relative rotation.

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These restrictions can be demonstrated by comparing the required position of the launch point for initiation of the mission as given by figure 6 with the actual position of the launch point as given by equation (1). Simultaneous solutions are the possible launch times. Equation (1) would be represented in figure 6 by a set of parallel, nearly vertical line segments. A few of these lines for an arbitrary phase angle are shown for illustrative purposes. The lengths of the line segments are chosen so that a line represents a change in launch-point location of 360°; thus, each line can be thought of as defining the possible launch times during a single day and the associated lunar positions.

For direct-ascent or constant-coast launches, ψ_r has a constant value. The intersections of a particular ψ_r-curve with the nearly vertical lines represent the possible launch times and lunar positions. For this type of launch, the possible lunar positions are represented by a discrete set of values spaced at about 13.2° intervals in ϕ_m. Since the position of the moon need not be defined any more accurately for successful completion of most missions, the relative rotation of the launch point and the moon do not seriously restrict the mission. However, only one launch is possible each day and if unexpected delays occur, which cannot be compensated by small changes in the dynamical variables, the launch must be postponed for about 24 hours, with a resulting movement of the moon from its design position. Some estimates of the launch-time tolerance can be found from the following considerations. Figure 4 indicates that at the nominal velocity ratio a change of ±100 fps $\left(\text{i.e., } \frac{V}{V_p} = \pm 0.003\right)$ will allow a change in ϕ_L of about $\pm 4^{\circ}$. This yields a launch-time tolerance of about ±16 minutes.

Parking- or coasting-orbit launches with a variable angular travel and launch heading angle remove this restriction. Figure 6 shows that almost all lunar positions are accessible and the launch time can be chosen at almost any time of the day for this type of launch. Of course, when any of the additional constraints discussed above are imposed on the launch conditions, the possible times of launch during the day are more restricted for all three types of launch trajectories. The restrictions on the launch time can be determined by comparing figure 6 with figures 3 or 5, depending on the type of constraint considered.

Injection-Point Locations

For any type of launch trajectory it is of interest to have the projection of the path and of the injection point on the surface of the rotating earth. In order to locate the path, it is assumed that between the times of launch and injection the vehicle has a mean geocentric angular velocity of $\frac{4}{3}\pi$ radians/hr, which corresponds to the rate of rotation of a satellite in a circular orbit at an altitude of about 170 miles. Figure 7 illustrates the path projections for a vehicle launched from Cape Canaveral with various heading angles. Small-circle arcs are drawn across the projections at 30° intervals to indicate the geocentric angular travel from the launch point to any point along the path. Also shown is the locus of injection points for hitting the moon at three declinations. There are two points on figure 7 where all of the path projections intersect. These points occur at the launch point and at the diametrically opposite point. It is noted that when a curve representing the injection-point locations passes through one of these points, there is a unique heading angle associated with this injection location; that is, these points should not be interpreted as injection positions where any launch heading is possible.

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The location of the injection points for velocity ratios other than 0.995 can be found by moving the injection point in figure 7 through an arc of $\Delta\psi$ taken from figure 4 at the appropriate values of injection angle and velocity ratio. The displacement along the path is in such a direction that a higher velocity ratio requires a longer coasting arc. If precession of the parking orbit is neglected, and since the earth rotates through 22.5° during one revolution of the vehicle, the location of the injection points for parking orbits can be found by displacing the points in figure 7 by 22.5° westward along a parallel of latitude for each revolution in orbit.

CONCLUDING REMARKS

By utilizing a simplified model with the moon considered to be a massless target, an analysis has been made of the restrictions imposed on the possible launch days and launch times of the day by the geometrical and dynamical conditions which must be specified for satisfactory completion of a lunar mission and some general remarks can be made.

For direct-ascent launch trajectories from the Atlantic Missile Range, the range azimuth safety restrictions seriously limit the lunar declinations which can be accommodated for most injection conditions. In general, this constraint requires that the approach of the vehicle to the moon be made during that part of the month when the moon is in the southern hemisphere. In addition, the time of launch during one of the possible launch days must be specified within fairly narrow limits about the design time.

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For properly chosen coasting arcs, the azimuth safety limits alone produce no restrictions on the possible launch days of the month for initiating coasting- and parking-orbit launches. If the launch system is designed so that the coasting-arc length and the launch azimuth can be varied throughout the day, then considerable freedom is allowed in choosing the daily launch time also.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., May 31, 1961.

APPENDIX

CALCULATION OF GEOMETRICAL PARAMETERS

The geocentric angular travel from the injection point $\left(\gamma,\ V/V_p,\ r_i\right)$ to the vicinity of the moon $\left(r_m\right)$ is given by the difference in the true anomalies of the two points and may be written

$$\psi_b = v_m - v_i \tag{A1}$$

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The two-body equations for the true anomalies are

$$\cos v_m = \frac{1}{e}\left[2\left(\frac{r_i}{r_m}\right)\cos^2\gamma\left(\frac{V}{V_p}\right)^2 - 1\right] \tag{A2}$$

$$\cos v_i = \frac{1}{e}\left[2 \cos^2\gamma\left(\frac{V}{V_p}\right)^2 - 1\right] \tag{A3}$$

The eccentricity is calculated from the equation for the conservation of angular momentum in the form

$$e = \sqrt{4 \cos^2\gamma\left(\frac{V}{V_p}\right)^2\left[\left(\frac{V}{V_p}\right)^2 - 1\right] + 1} \tag{A4}$$

In deriving analytical expressions between the angular parameters it is convenient to use the declination to specify the position of the moon. Then angular position as shown in figure 1 is related to the declination by

$$\sin \phi = \tan \delta \cot i \tag{A5}$$

The total angular arc from launch to the vicinity of the moon can be calculated from

$$\sin \psi_t = \frac{\sin^2\delta - \sin^2\lambda}{\cos \lambda \sin \delta \cos \beta + \sin \lambda \cos \delta \cos \rho} \tag{A6}$$

and

$$\cos \psi_t = \frac{\sin \delta - \cos \lambda \cos \beta \sin \psi_t}{\sin \lambda} \tag{A7}$$

where the heading of the vehicle in the vicinity of the moon is given by

$$\sin \rho = \frac{\cos \lambda \sin \beta}{\cos \delta} \qquad (0 \leqq \rho \leqq \pi) \tag{A8}$$

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For each heading angle there are, in general, two values of ψ_t corresponding to the two points where a great circle intersects a parallel of latitude. These are calculated by using the two roots of

$$\cos \rho = \pm\sqrt{1 - \sin^2\rho} \tag{A9}$$

in the equation for $\sin \psi_t$. After ψ_b is calculated from the two-body equations and ψ_t from the geometrical equations, the required arc from the launch point to the injection point is the difference

$$\psi_s = \psi_t - \psi_b \tag{A10}$$

The angle between the earth-moon and earth-vehicle planes is given by

$$\alpha = \rho - \eta \tag{A11}$$

where ρ is given by equation (A8) and η is calculated from

$$\cos \eta = -\sin i \cos \phi \qquad (0 \leqq \eta \leqq \pi) \tag{A12}$$

The required angular position of the launch point is given by

$$\phi_L = \phi_m - \theta \tag{A13}$$

where θ can be calculated from

$$\cos \theta = \frac{\cos \psi_t - \sin \lambda \sin \delta}{\cos \lambda \cos \delta} \tag{A14}$$

and ψ_t is given by equations (A6) and (A7).

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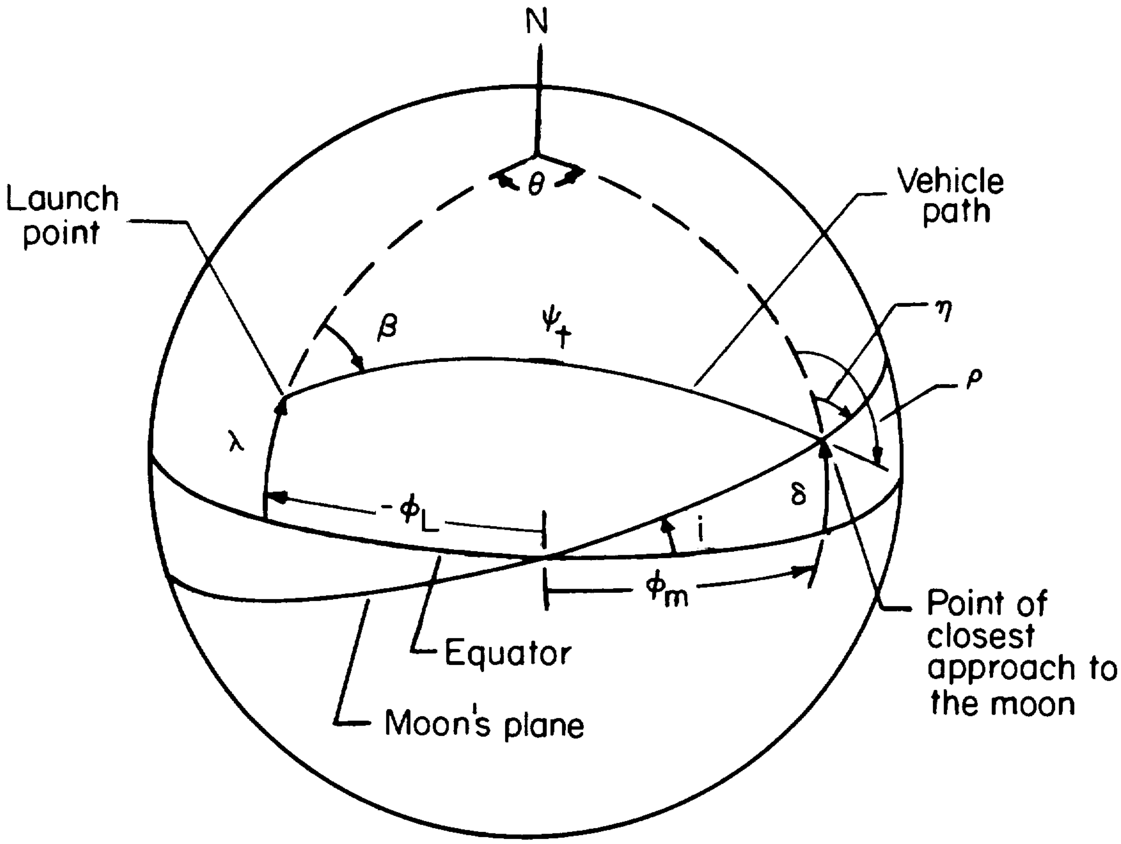
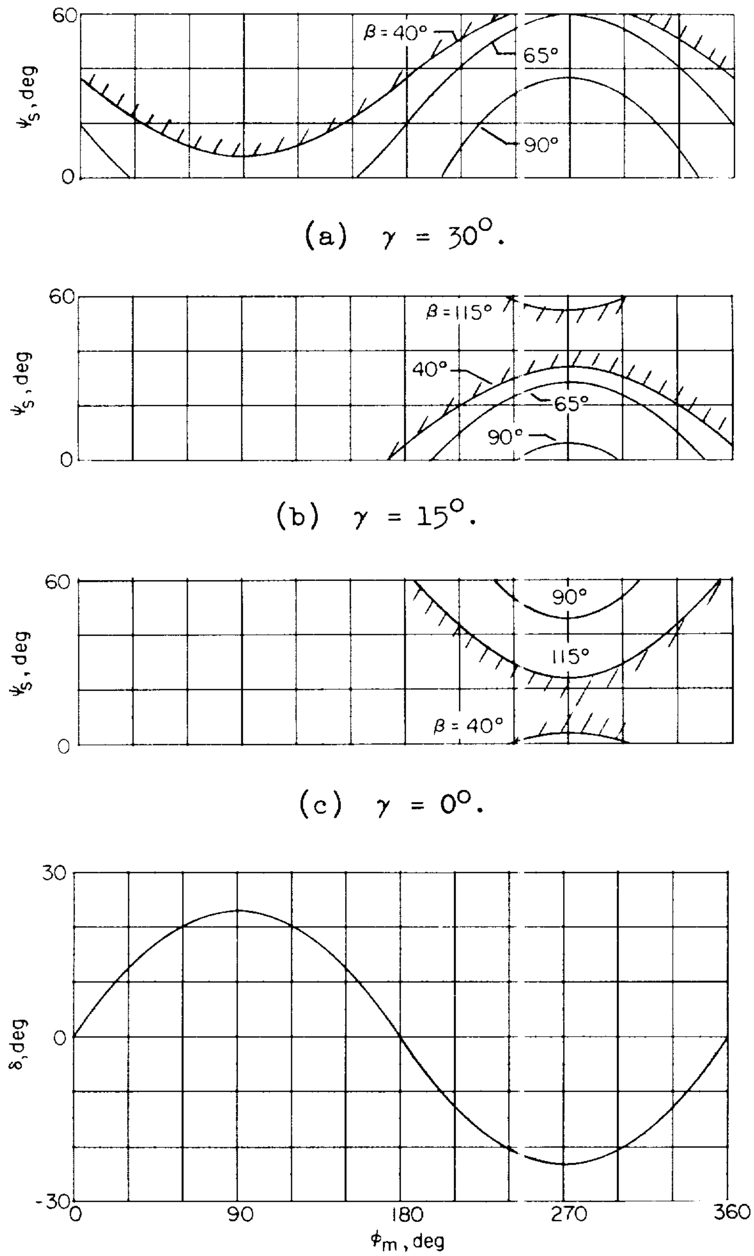


Figure 1.- Illustration of some angular parameters.



(d) Lunar declination.

Figure 2.- Possible lunar positions for direct-ascent launch trajectories. $\frac{V}{V_p} = 0.995$.

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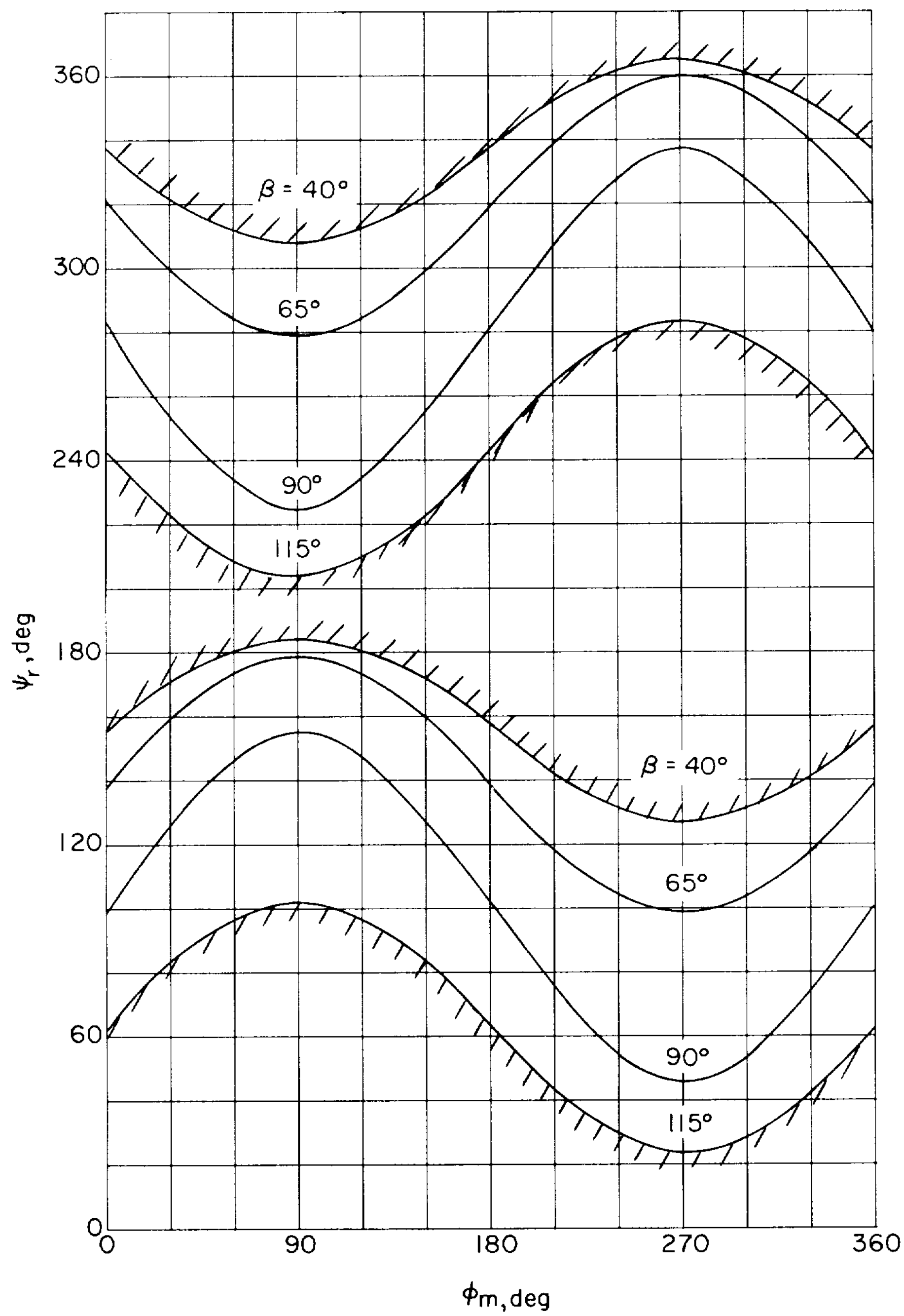

Figure 3.- Lunar position which can be accommodated for coasting-orbit launches. $\frac{V}{V_p} = 0.995$.

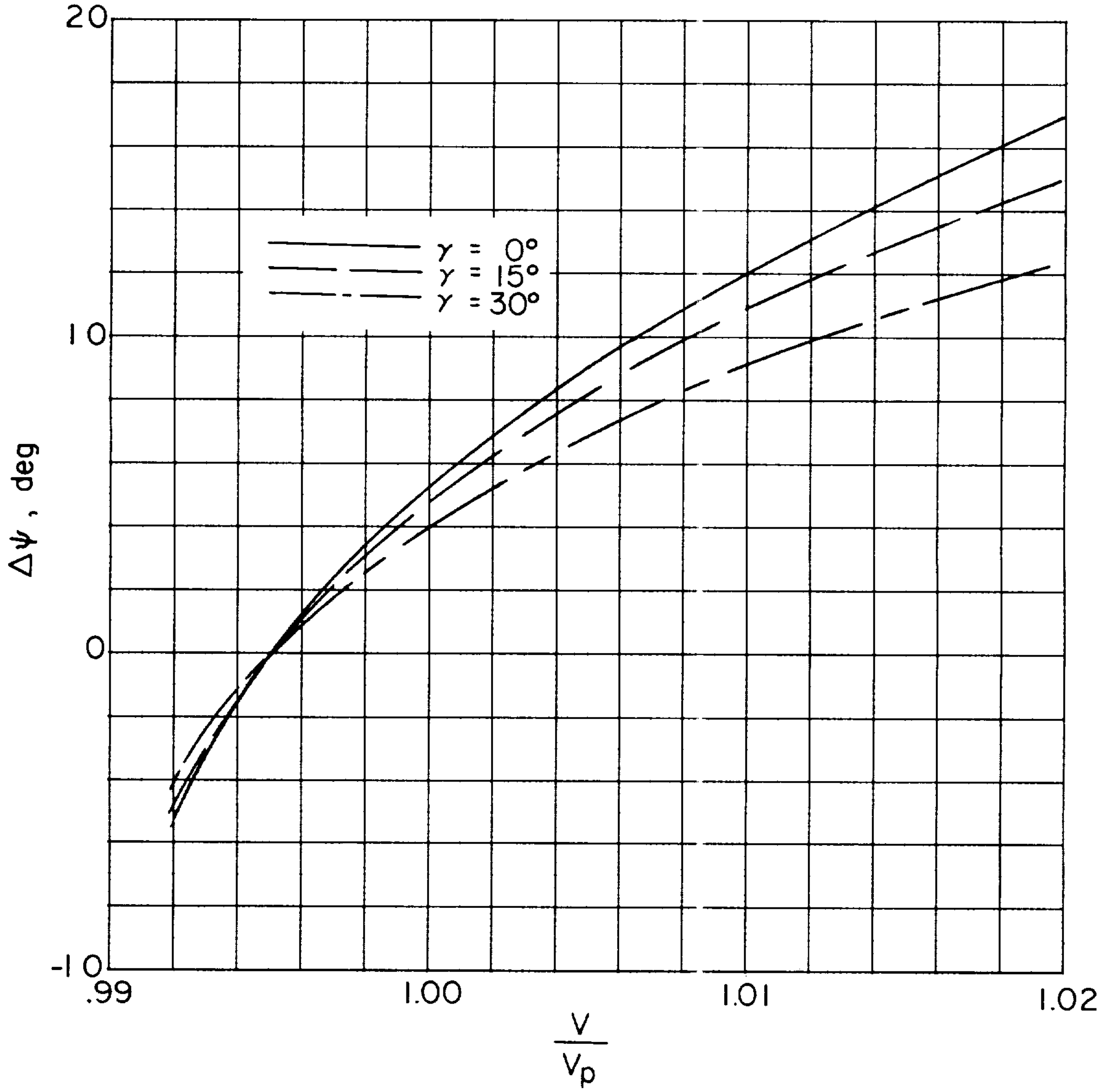

Figure 4.- Angular travel corrections for changes in injection velocity ratio.

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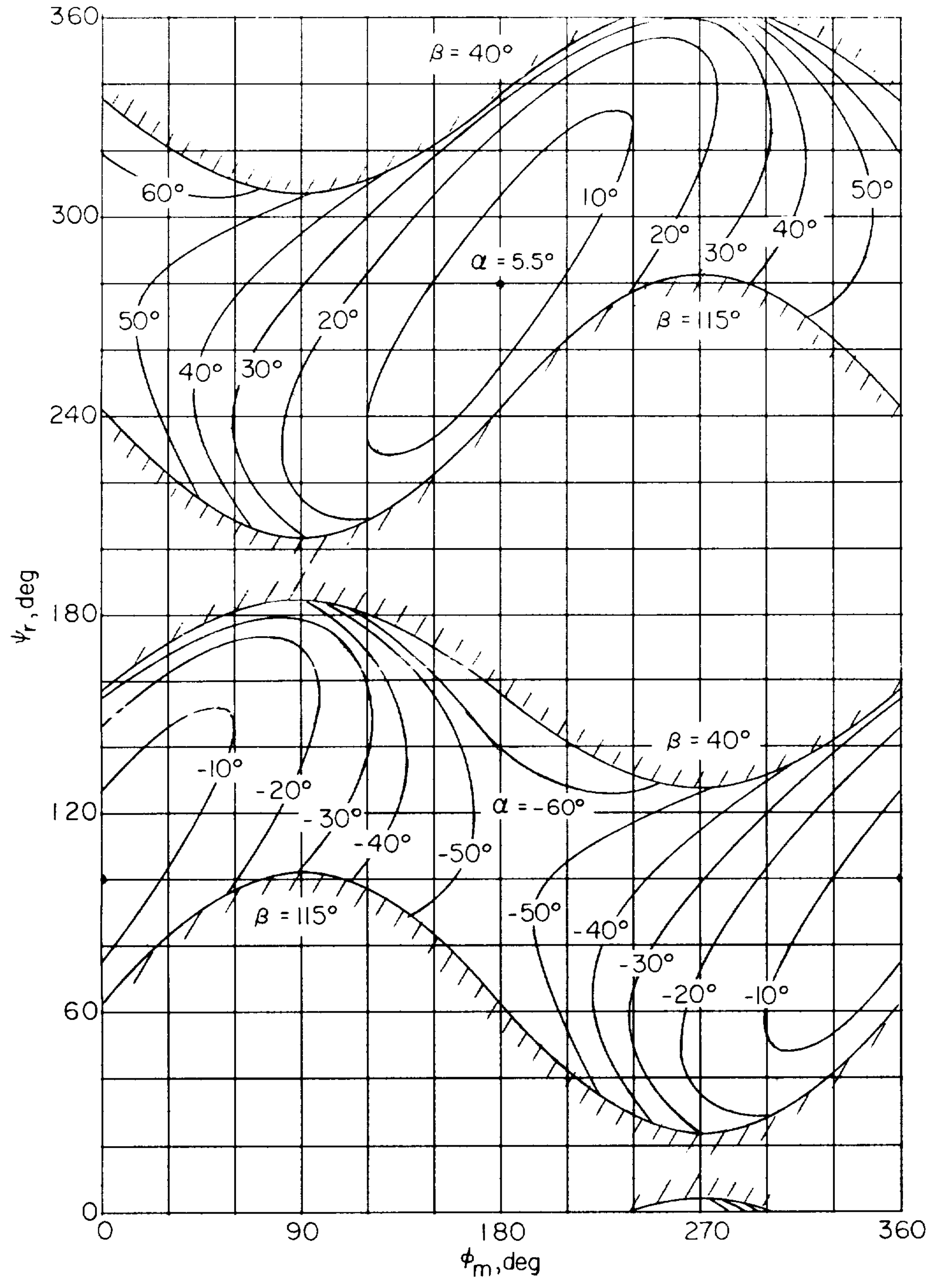


Figure 5.- Variation of initial angle between the earth-moon plane and the vehicle-trajectory plane with lunar position and reduced angular travel.

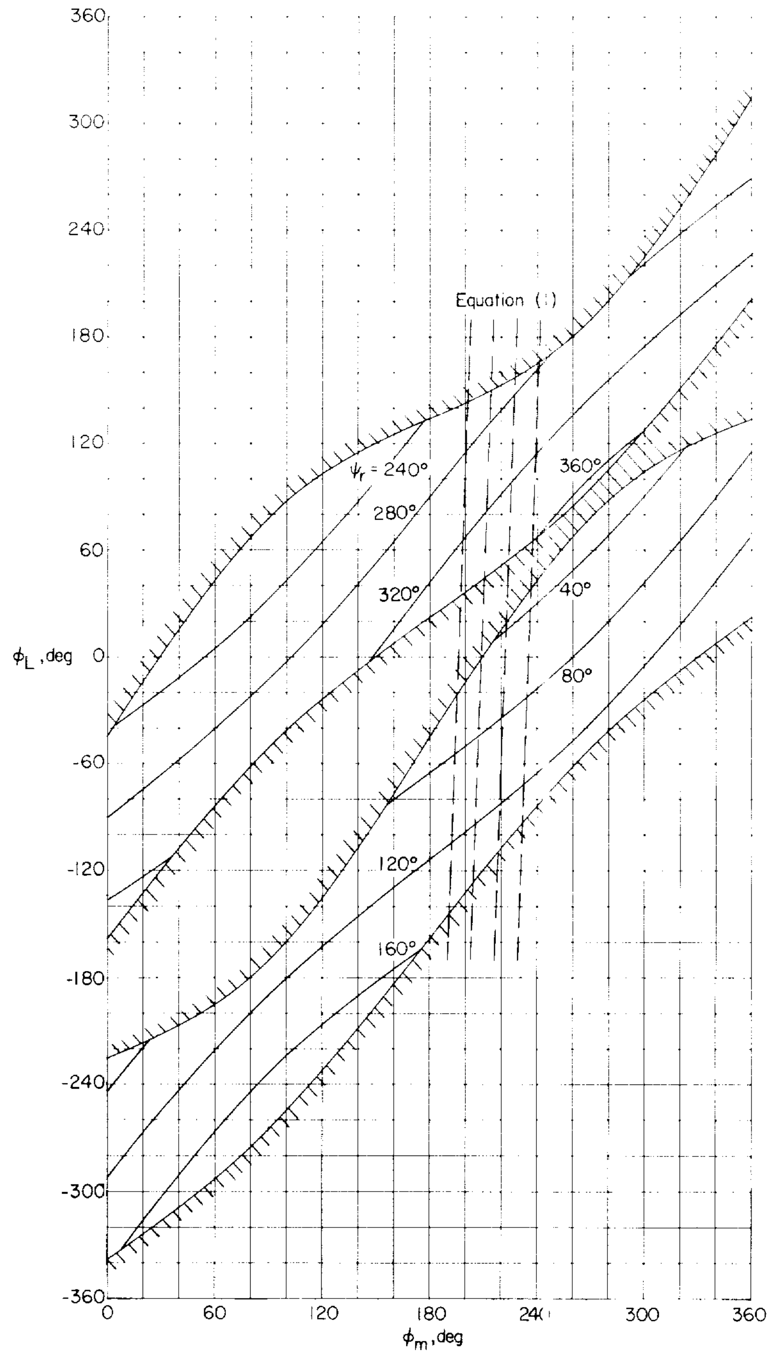
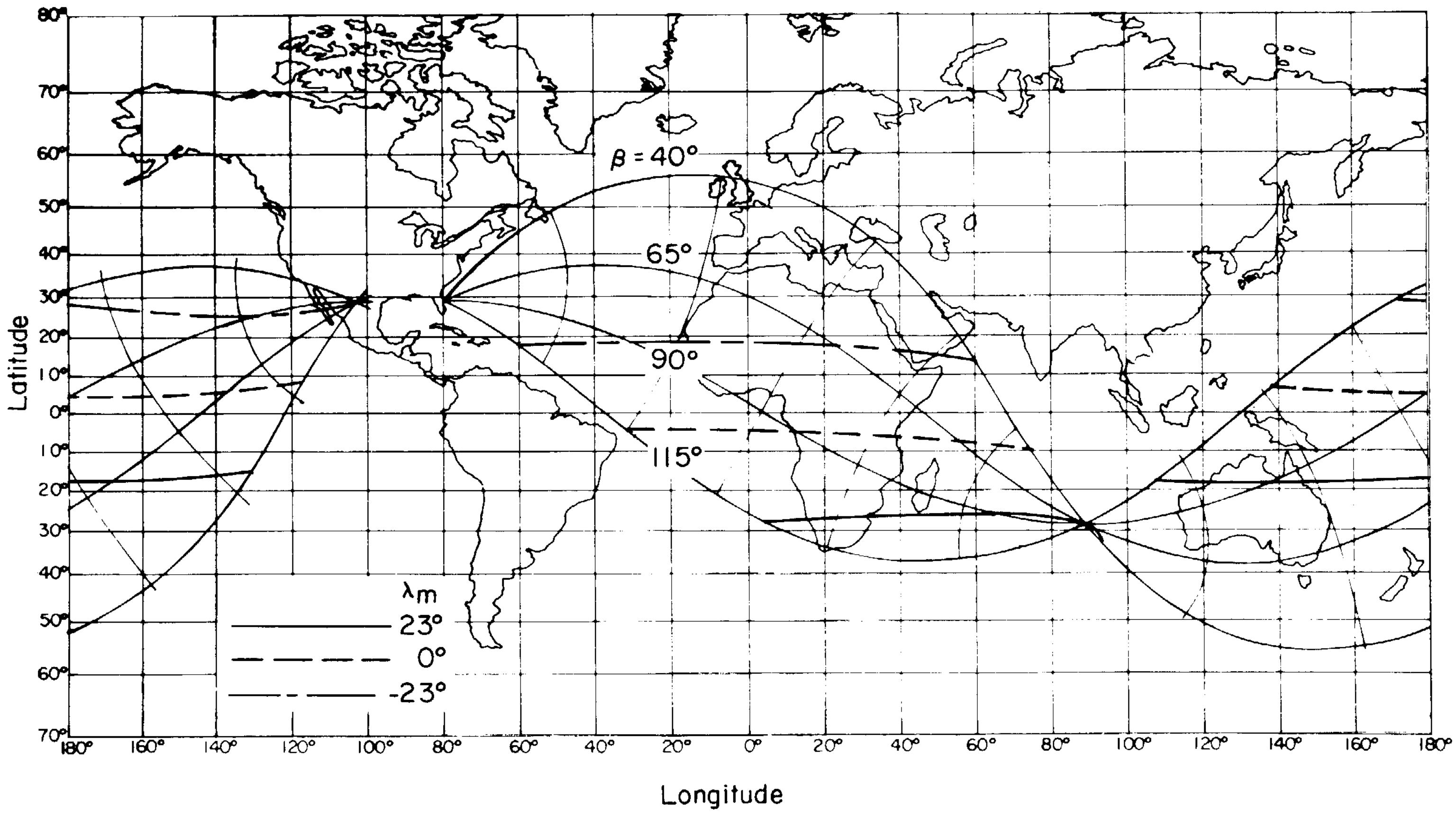


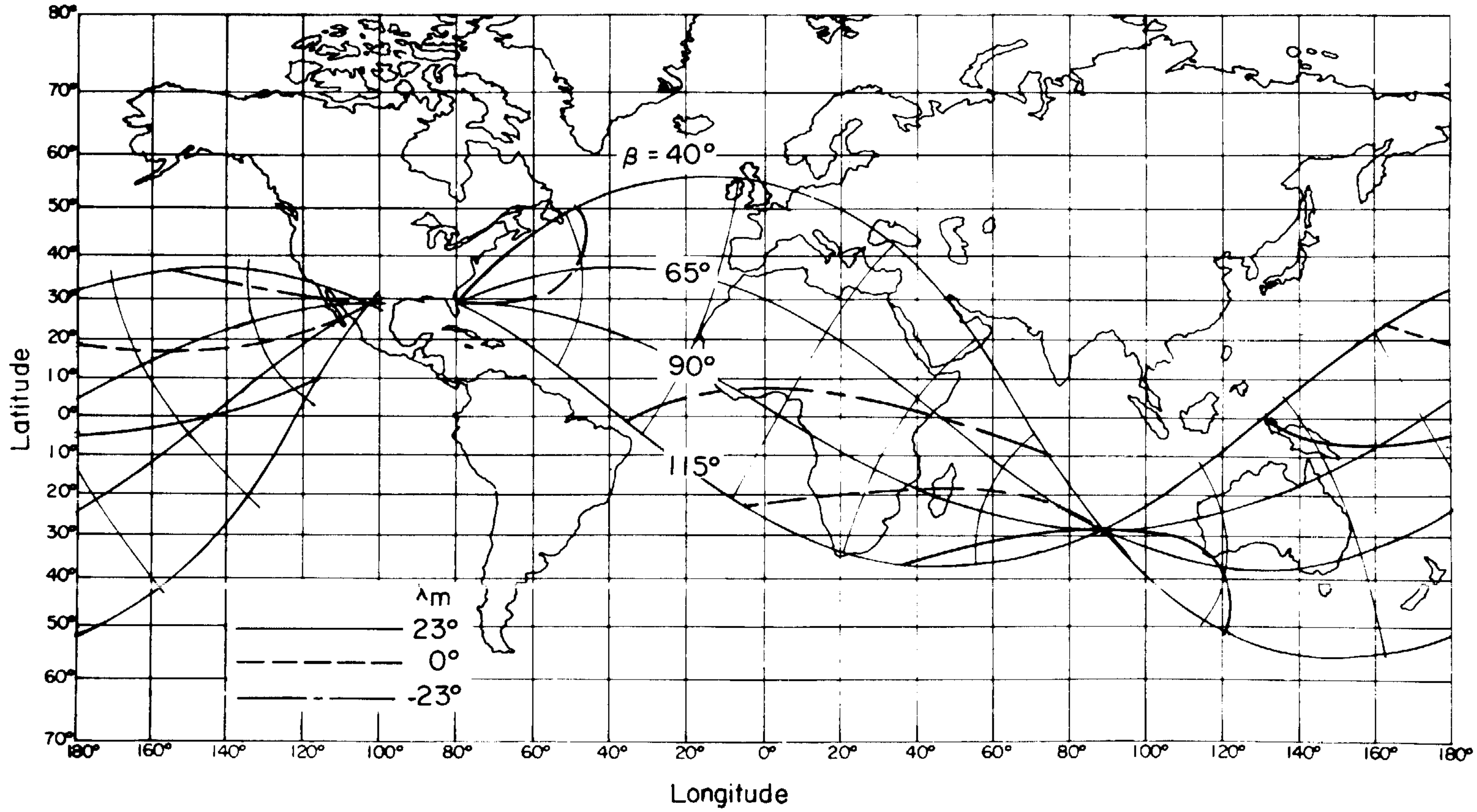
Figure 6.- Required launch-point position as a function of lunar position and reduced angular travel.

L-1517



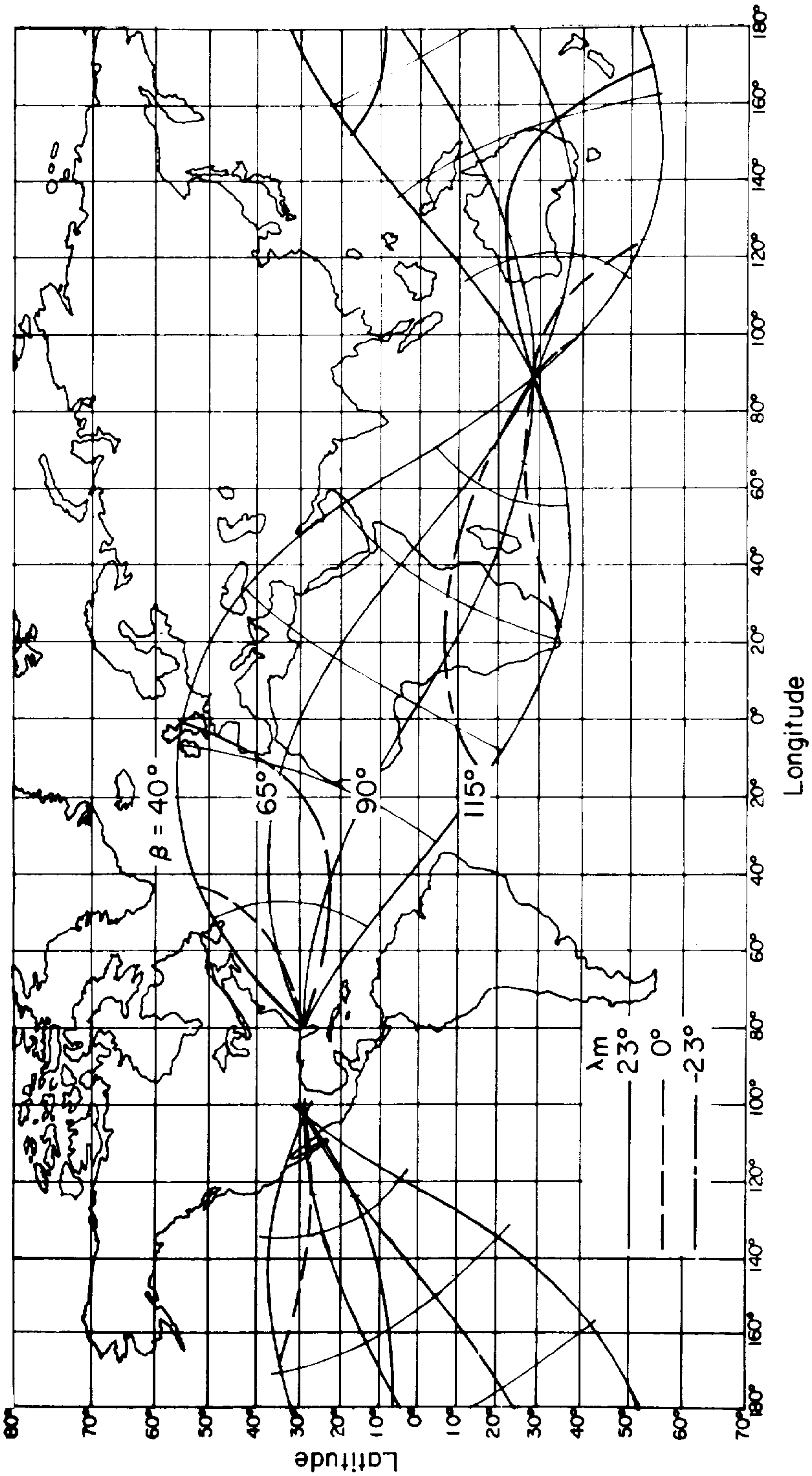
(a) $\gamma = 0^{\circ}$.

Figure 7.- Injection-point locations for approaching the moon at various declinations.



(b) $\gamma = 15^{\circ}$.

Figure 7.- Continued.



(c) $\gamma = 30^{\circ}$.

Figure 7.- Concluded.